LA-UR-21-24337

Title: Information Scrambling in Complex Quantum Systems

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Intended for: Report

Issued: 2021-05-05

LA-UR

Information Scrambling in Complex Quantum Systems

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CNLS, 05/14/2021

Theoretical Division Since 1943

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Center for Nonlinear Studies

Quantum information/Statistical physics

Information Scrambling : Rapid spreading of local information over the entire physical system.

- Many-body quantum systems; complex (chaotic) dynamics
- Entanglement generation
- Universal characteristics

Out-of-time order correlator (**OTOC**)

Quantum butterfly effect I:

Bin Yan, L. Cincio, and W. H. Zurek, “Information Scrambling and Loschmidt Echo.” *Physical Review Letters* **124** (16): 160603 (2020).

Quantum butterfly effect II:

Bin Yan, and N. A. Sinitsyn. “Recovery of Damaged Information and the Out-of-Time-Ordered Correlators.” *Physical Review Letters* **125** (4): 040605 (2020).

N. A. Sinitsyn, **Bin Yan**, “The quantum butterfly non-effect.” *Scientific American*, September 2020.

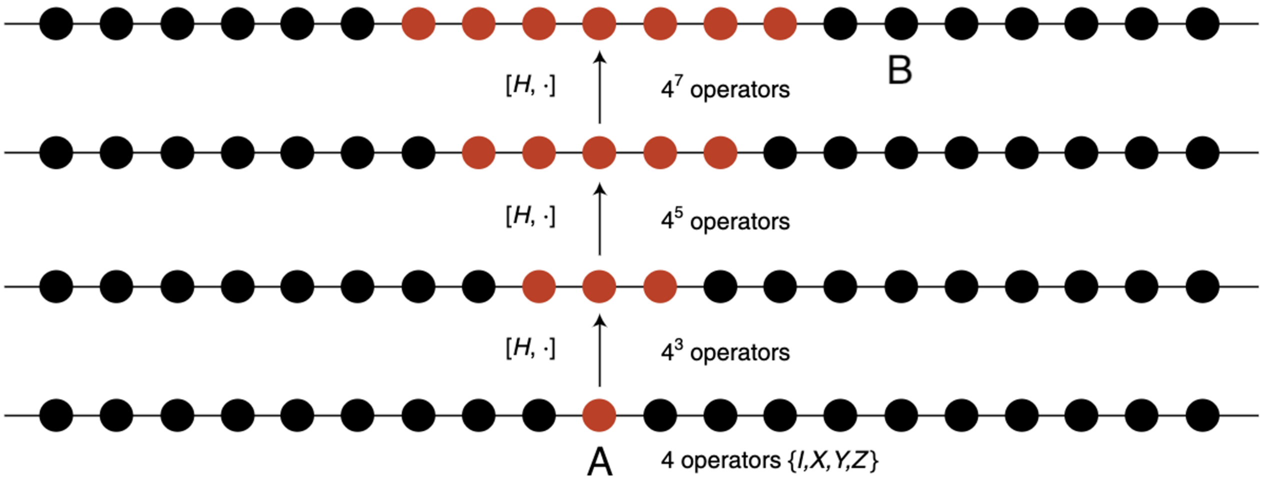
Random unitaries (2-design):

Z. Holmes., A. Arrasmith, **Bin Yan**, and P. Coles, A. Albrecht and A. Sornborger, “Barren Plateaus Preclude Learning Scramblers.” *Physical Review Letters* (2021), in press; co-first author.

Out-of-time order correlator (OTOC) :

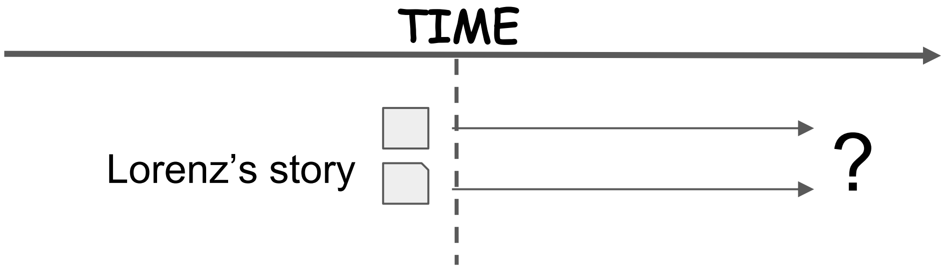
$$\boxed{\langle\Psi|A(t)BA(t)B|\Psi\rangle}, \quad A(t) = U(t)AU^\dagger(t)$$

$$= 1 - \langle\Psi|[A(t), B]^2|\Psi\rangle/2$$



[1] Larkin, A, and Ovchinnikov, Y. N., Soviet Physics JETP, **28**, 1200 (1969).
[2] Kitaev, A, (2015) KITP talk: "A simple model of quantum holography."
[3] Swingle, B., Nature Physics **14** (10): 988–90 (2018).

Lorenz butterfly effect



Edward Lorenz (1972): Does the flap of a butterfly's wings in Brazil set off a tornado in Texas?

$$|\psi_1(t)\rangle = e^{-iHt}|\psi_0\rangle$$
$$|\psi_2(t)\rangle = e^{-iHt}|\psi_0\rangle'$$
$$\langle\psi_1(t)|\psi_2(t)\rangle = \langle\psi_0|\psi_0\rangle' = \text{constant}$$

No quantum butterfly effect?

- Quantum chaology

Berry, M. Quantum chaology, not quantum chaos. Phys. Scr. ***40****, 335 (2006)*

$$|\psi_1(t)\rangle = e^{-iHt}|\psi_0\rangle$$
$$|\psi_2(t)\rangle = e^{-i(H+V)t}|\psi_0\rangle$$
$$\langle\psi_1(t)|\psi_2(t)\rangle = \langle\psi_0|e^{i(H+V)t}e^{-iHt}|\psi_0\rangle$$

There is quantum butterfly effect

- Loschmidt echo

Quantum Theory: Concepts and Methods. A. Peres (Springer, 2002).

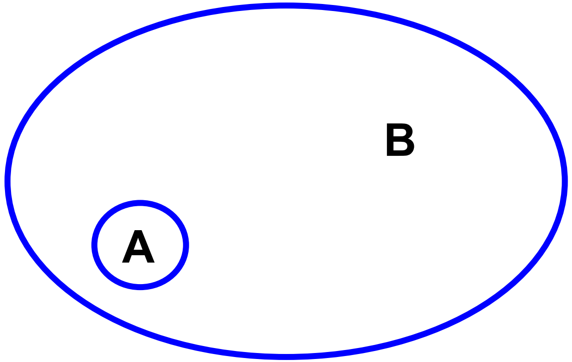
$$\overline{\mathrm{OTOC}} \equiv \int dA dB \; \mathrm{Tr}\left[A(t)BA(t)B\rho\right]$$

$$H = \mathbb{I}_A \otimes H_B + H_A \otimes \mathbb{I}_B + H',$$

$$H' \equiv \delta \sum_k V_A^k \otimes V_B^k.$$

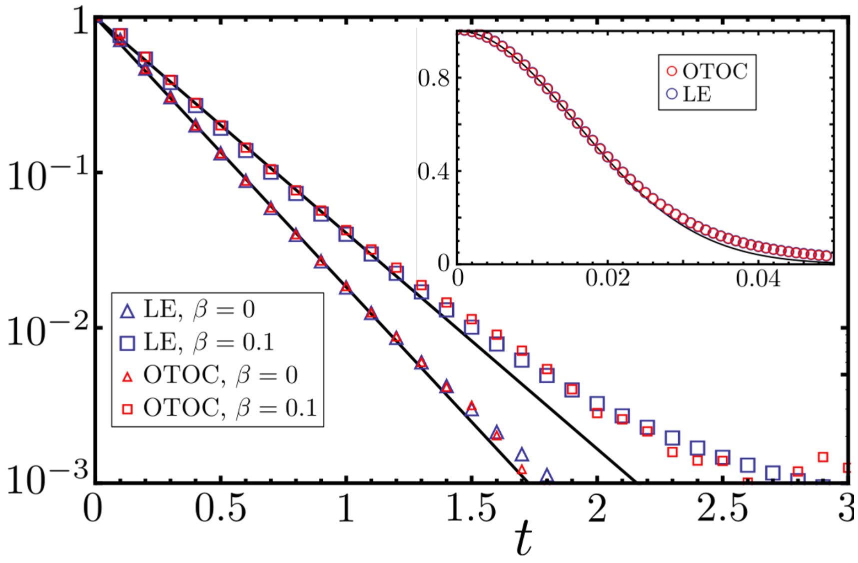
$$\overline{\mathrm{OTOC}} \approx |\langle e^{i(H_B+V)t} e^{-iH_B t}\rangle|^2$$

$$V = \delta \overline{V_B^k}$$



Bin Yan, L. Cincio, and W. H. Zurek, *Phys. Rev. Lett.* **124** (16): 160603 (2020).

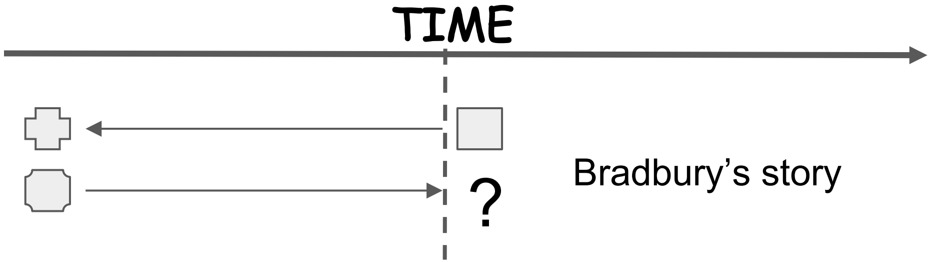
1. Random matrix model
2. Exactly solvable coupled harmonic oscillators
3. Sachdev-Ye-Kitaev (SYK) model



- H_A, H_B, H_I are all random matrices
- Strong interaction: Gaussian decay
- Weak interaction: Exponential decay

Bin Yan, L. Cincio, and W. H. Zurek, *Phys. Rev. Lett.* **124** (16): 160603 (2020).

Bradbury butterfly effect



Bradbury’s story

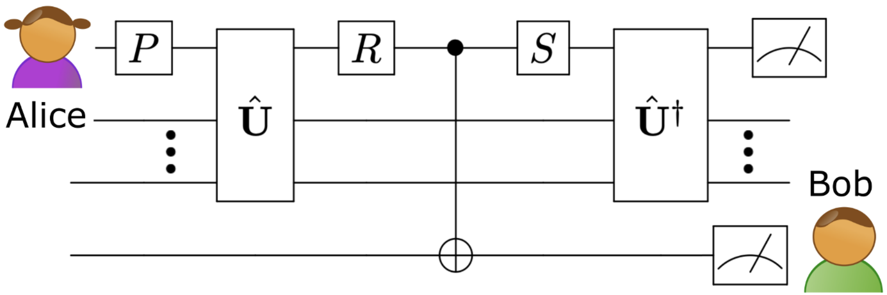
Ray Bradbury (1955): Science fiction “A sound of thunder”

$$\mathrm{Tr}\left[A(t)BA(t)B\rho\right]$$
$$=\mathrm{Tr}\left[U^\dagger AUBU^\dagger AUB\rho\right]$$

OTOC is the Bradbury’s butterfly

State evolution through the Bradbury process averaged over all random unitaries.

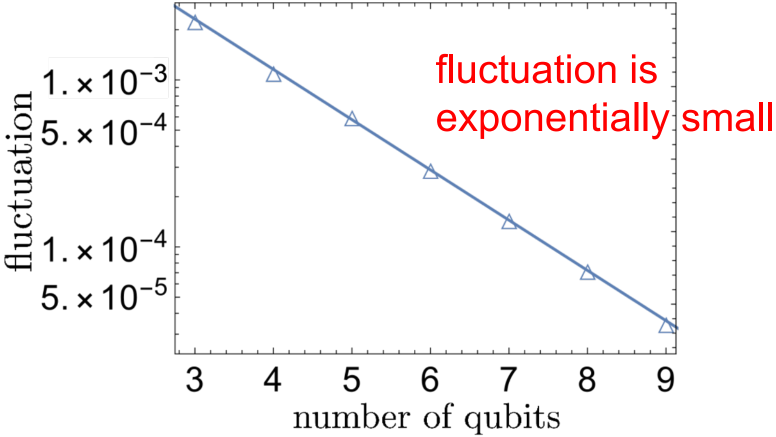
Bin Yan, and N. A. Sinitsyn, *Phys. Rev. Lett.* **125** (4): 040605 (2020).

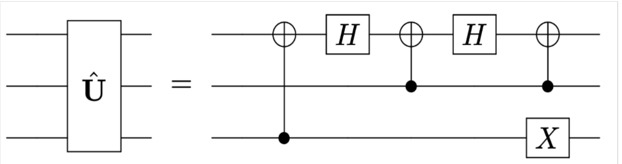


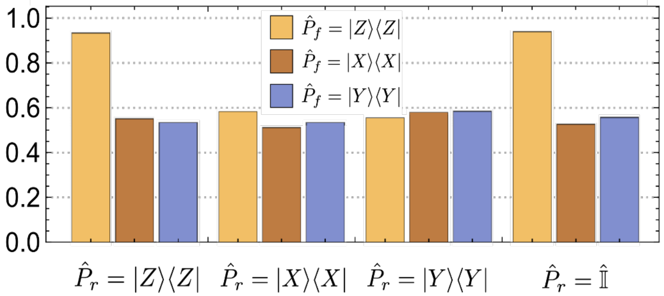
- Alice prepares an a initial state; Goes through a Bradbury process; Final state ?
- Partial information of Alice’s qubit always comes back:

$$\hat{\rho}_{\mathrm{out}} = \frac{1}{2}\hat{\rho}_{\mathrm{in}} + \frac{1}{4}\hat{\mathrm{I}}$$

- Averaged behavior represents the typical behavior.

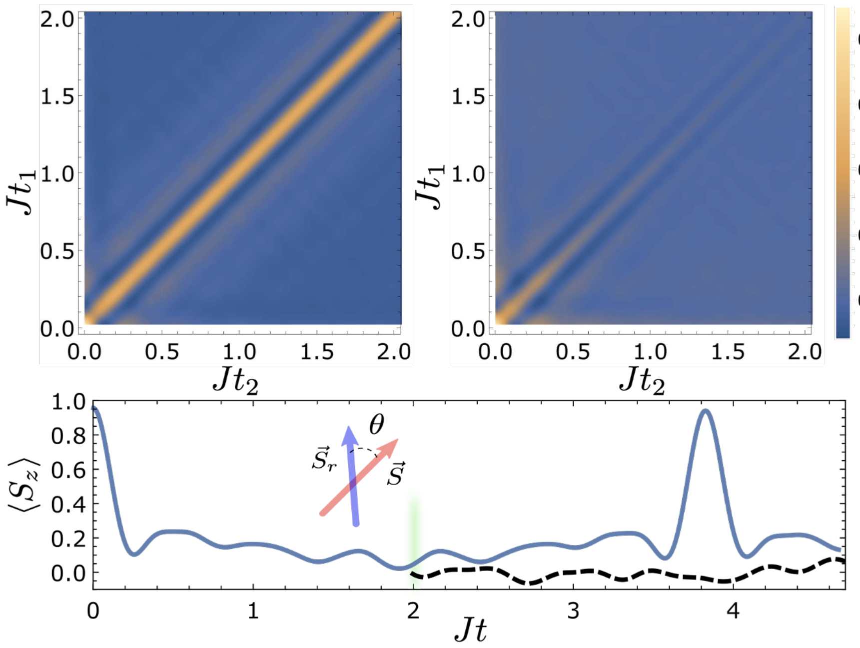






Physical model : spin-bath

$$H = \sum_{i=1}^{N} \sum_{\alpha} J_i^{\alpha} S^{\alpha} s_i^{\alpha}, \ \alpha = x, y, z,$$



- t1 forward evolution
  t2 backward evolution
- Top-left: recovery signal
  Top-right: spin echo
- Buttom: classical analog

anti-butterfly effect

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The Quantum Butterfly Noneffect

Scientific American, 21 Sep 2020

Chaos theory says that a tiny, insignificant event or

The Economist

News story from The Economist

The Economist, 13 Aug 2020

DISCOVER MAGAZINE

Does the Butterfly Effect Exist?

Discover Magazine, 17 Aug 2020

In “A Sound of Thunder,” the short story by

•

•

LANL bi-annual scientific magazine **1663**
Cover story : The quantum butterfly effect

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OTOC, random unitary and quantum machine learning

$U(t) = e^{-iHt}$ is complex (random) for large times

unitary k-design

$$\rho \to \Lambda_{\mathcal{E}}(\rho) = \int_{\mathcal{E}} dU \; (U^\dagger)^{\otimes^k} \rho U^{\otimes^k}$$

$$\rho \to \Lambda_{\mathrm{Haar}}(\rho) = \int_{\mathrm{Haar}} dU \; (U^\dagger)^{\otimes^k} \rho U^{\otimes^k}$$

$$\Lambda_{\mathcal{E}}(\rho) = \Lambda_{\mathrm{Haar}}(\rho) \quad \forall \rho$$

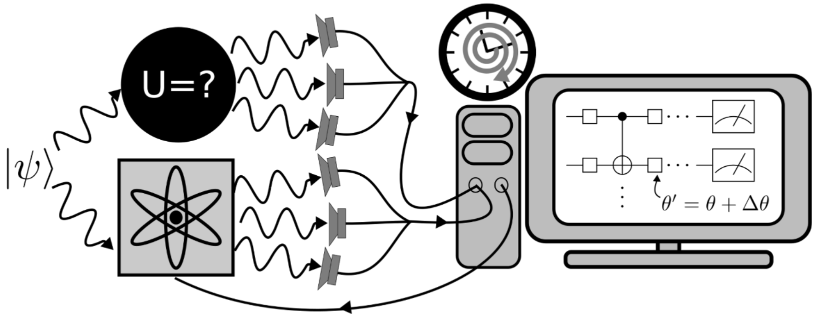
$$\int_{\mathcal{E}} dU \; \mathrm{Tr}\left[U^\dagger A U B U^\dagger C U D\right] = \int_{\mathrm{Haar}} dU \; \mathrm{Tr}\left[U^\dagger A U B U^\dagger C U D\right]$$

$$\forall A, B, C, D$$

4-point OTOC detects unitary 2-designs

A scrambler is a unitary evolution U(t) that becomes at least a 2-design

Quantum machine learning cannot effectively learn a scrambling physical process. [1]



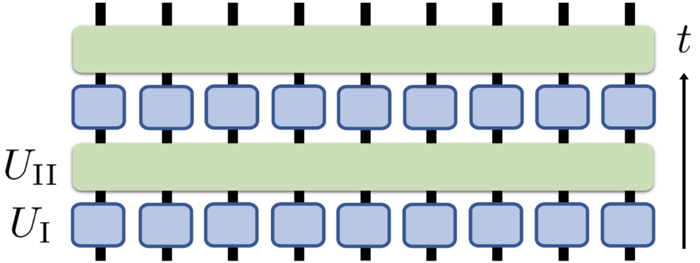
$$C(\theta, U_T)$$
$$= \langle\psi|V(\theta)^\dagger U_T H U_T^\dagger V(\theta)|\psi\rangle$$

$$\langle \partial_\theta C(\theta, U_T)\rangle_{\mathcal{E}} = 0$$

Barren plateau

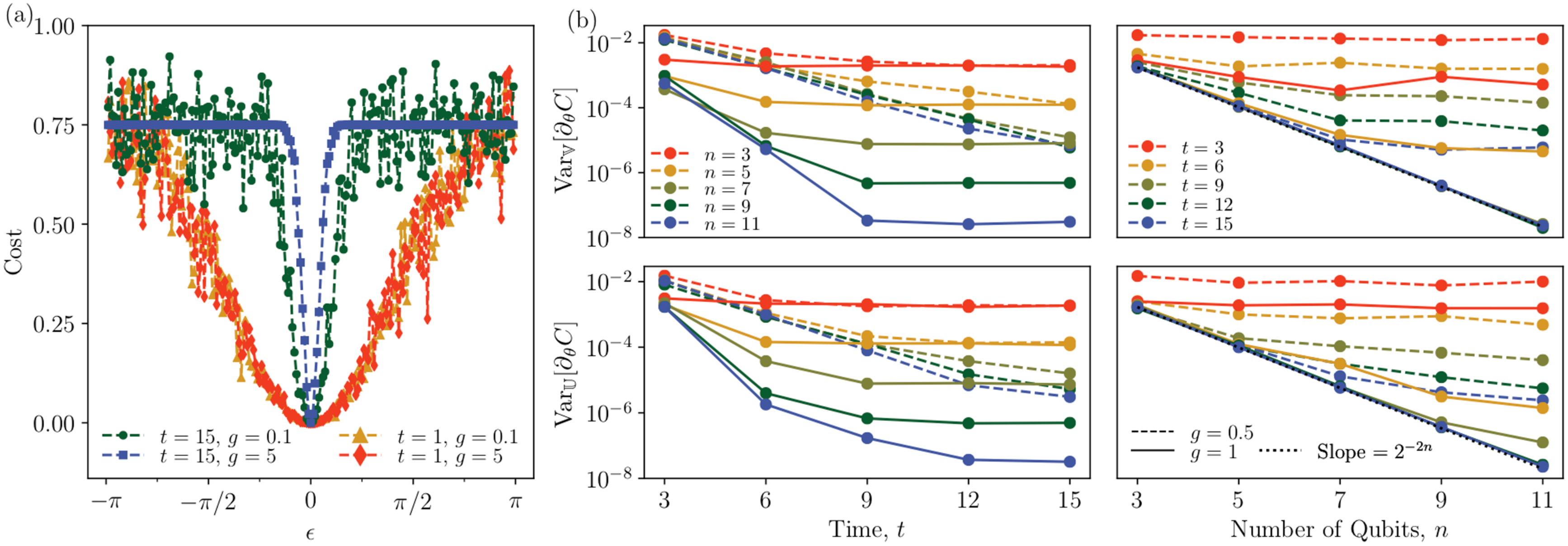
$$\mathrm{Var}_{\mathcal{E}}\left[\partial_\theta C(\theta, U_T)\right] = \mathcal{O}(2^{-n}),$$
$$\mathrm{Var}_{\mathcal{E}} \equiv \langle(\partial_\theta C(\theta, U_T))^2\rangle_{\mathcal{E}} - \langle\partial_\theta C(\theta, U_T)\rangle_{\mathcal{E}}^2$$

Z. Holmes., A. Arrasmith, **Bin Yan**, *at al*, arxiv : 2009.14808 (2020). *Phys. Rev. Lett.* in press;



$$U(t) = (U_{II} U_I)^t$$

$$U_{II} = e^{-i \frac{g}{2\sqrt{N}} \sum_{i<j} Z_i Z_j}$$



Z. Holmes., A. Arrasmith, **Bin Yan**, *at al*, arxiv : 2009.14808 (2020). *Phys. Rev. Lett.* in press;

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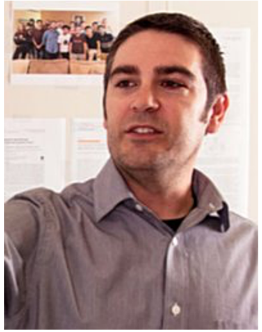

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